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ABSTRACT

A specific Normative Location Theory procedure, called hyperbolic approximation (HAP), is suggested as a possible "new" initial-configuration strategy for multidimensional scaling in the city-block metric. First, the performance of this strategy was investigated using fourteen simulated data sets. Second, the scaling in Euclidean space of actual data that have underlying city-block structure was explored. Although HAP is comparable to existing procedures for generating initial configurations for multidimensional scaling, the scaling results of the simulated data reverified that the city-block metric is prone to local minima. The conflicting results within the second investigation suggest that further study is needed. (Author)

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THE USE OF THE CITY-BLOCK METRIC IN MULTIDIMENSIONAL SCALING

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Paper presented at AERA Convention, New York

Since Shepard's introduction of a computational procedure for nonmetric multidimensional scaling (NMDS) and Kruskal's contribution of an efficient algorithm, the use of NMDS has spread to areas other than psychology; for example, Green and Carmone (1972) give applications to marketing analysis, and Subkoviak (1975) gives applications in education. The major purpose of NMDS in all of its applications is to provide a representation for the underlying structure of a data set in a visual form such as a geometric model or picture; consequently, the choice of an appropriate form for the underlying metric has become an important issue.

Although Shepard's original method of scaling allowed the fitting of configurations that were based on metrics other than the Euclidean, the Euclidean metric is by far the most commonly used distance function. A substantial amount of empirical evidence now exists, however, suggesting that psychological proximities may at times be better represented by a nonEuclidean measure. For example, Householder and Landahl (1945), Landahl (1945), and Young and Householder (1938) proposed a scaling method based on a priori knowledge of the dimensions on which the entities were to be scaled. From a neurophysical foundation, they assumed that the discrimination between two stimuli was governed by the city-block-distance function. Attneave (1950) developed this same idea in more detial and showed how the city-block norm was applicable to psychological judgments of similarity as well. In the city-block model, the distance between two points is defined by the total distance that must be traversed in the north-south direction plus the distance that must be traversed in the east-west direction; consequently, one of the main features of this model from a psychological point of view is that similarities are judged along dimensional axes, which are regarded as fixed and not subject to rotation as in the Euclidean model. Torgerson (1958), Attneave (1950), and Shepard (1964) all suggest that the nature of the stimulus objects is related to the type of spatial model that will best fit the data. In particular, the city-block metric may be required for stimuli, like geometrical figures, that tend to be analyzed into their values along certain preferred reference axes that are not subject to arbitrary rotation. Alternatively, the Euclidean metric may be best for

stimuli, like colors, that tend to be reacted to as homogeneous, unanalyzable wholes. Hyman and Well (1967, 1968) have provided experimental confirmation of the perceptual-separability hypothesis by demonstrating that the city-block metric is most appropriate when the dimensions accounting for object differences are clearly defined and when subjects sum object differences across each of the separate dimensions.

Methods of NMDS are suited to the study of alternative Euclidean or non-Euclidean rules of combination of differences along underlying psychological dimensions (for example, see Arnold, 1971; Behrman & Brown, 1969; Brown & Andrews, 1968; Brown & Brumaghin, 1969; Lund, 1970; Rapoport & Fillenbaum, 1972; Stenson, 1968; Thomas, 1968). Generally, the way investigators determine which metric best explains the underlying psychological structure is by seeking Minkowski metric representations for various values of r and then choosing one best solution, usually on the basis of lowest residual stress (Kruskal, 1964; p. 24).[1] The most commonly used metrics are the city-block (r=1), the Euclidean (r=2), and the dominance metric (r=∞). Even though this selection strategy is standard, Shepard (1974, pp. 401-2) claims that the procedure is open to serious criticism:

> This practice of taking, as the correct metric, the one that yields the lowest residual stress is unfounded and probably leads to erroneous conclusions. Such a practice is based on the assumption, never explicitly justified, that values of stress are directly comparable across different values of r. From the practical standpoint, the cost of obtaining solutions separately for many different values of r is apt to become prohibitive because problems of slow convergence and local minima are much more severe in the case of these non-Euclidean metrics.

In all of the above studies where investigators explicitly compared Euclidean and city-block stress values, the Euclidean solution had the lower value; consequently it was chosen as the best representation of the underlying psychological structure. The various psychological theories, on the other hand, had postulated a city-block metricfor explaining the psychological structure of the data, and futhermore, in those studies in which the investigator tried to determine what metric best represented the data by methods <u>other</u> than NMDS, the results conformed to theoretical predictions. That is, for analyzable percepts a city-block model was appropriate, whereas for non-

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[1] r is the value in the general Minkowski r-metric distance function in m dimensions defined by

$$d_{ij}^{(r)} = \left[\sum_{k=1}^{m} |x_{ik} - x_{jk}|^r \right]^{1/r},$$

where x_{ik} is the coordinate value for the i^{th} object on the k^{th} dimension.

analyzable percepts, the Euclidean model provided the best representation (for example, see Attneave, 1950; Handel & Imai, 1972; Hyman & Well, 1967, 1968; Shepard & Cermack, 1973; Waern, 1969). A general difficulty still exists, however, in trying to develop reasonable indices of fit, and as Sherman (1972, p. 354) concluded, based on his observation of the effect of different parameters on the adequacy of the scaling solutions, "Given the hypothesized existence of a true underlying configuration, stress is far from a perfect indicator of true fit, but it is the best available and can be better interpreted with supplementary information." Perhaps the answer does not lie in obtaining a better index of fit that would be comparable across all metrics, but in alternative initial-configurations for multidimensional-scaling programs that deal with the city-block metric alone. Hopefully, this strategy would alleviate the problem of beingtrapped in local minima, because its justification does not depend on the traditional factor-analytic concepts that form the basis for existing NMDS initialization techniques.

The task of determining an appropriate form for the underlying metric of an m-dimensional placement was approached by investigating the utility of alternative initial-configuration schemes in conjunction with the city-block metric. A specific Normative Location Theory (NLT) procedure, called hyperbolic approximation (HAP), was suggested as a possible "new" initial-configuration strategy. In general, NLT deals with the placement of objects in a continuous space and requires that a certain set of objects (the fixed objects) be located a priori. The placement of the second set of objects (the variable objects) is then determined in such a way that the total "interaction" among these two sets of objects is minimized. Specifically, in the NLT framework, there are two sets of objects S_1 and S_2, and a nonnegative real-valued proximity function $s(\cdot,\cdot)$ defined on $(S_1 \cup S_2) \times (S_1 \cup S_2)$. Each object, say O_i, in $S_1 \cup S_2$ to be placed in m-dimensional space with coordinates $\underset{\sim}{x}_i = (x_{1i},\ldots,x_{mi})$, and in addition, a nonnegative real-valued function $d(\cdot,\cdot)$ is defined on pairs of such coordiante vectors. Given this framework, it is assumed that the coordinate vectors $\underset{\sim}{x}_{n+1},\ldots,\underset{\sim}{x}_{n+m}$ for the objects in S_2 are "fixed" and the goal is to find $\underset{\sim}{x}_1,\ldots,\underset{\sim}{x}_n$ that minimizes the NLT objective function:

$$\sum_{j=1}^{n+m} \sum_{i=1}^{n+m} s(O_i,O_j)d(x_i,x_j).$$

In the proposed procedure, S_1 and S_2 are usually assumed to be the same; thus the common object set operates alternatively as a fixed set and as a variable set.

In order to investigate the performance of the NLT strategy when used in conjunction with NMDS, configurations with a recognizable form consisting of ten

(a kite shape) and twenty-five (a circular shape) points were constructed in two-dimensional space. The ten-point configuration was replicated three times. In particular, fourteen independent sets of similarity data were simulated for the two configurations. In addition, seven sets of data were generated using the Euclidean distance function for the ten-point configuration, which was also replicated three times. These initial figures were called the "true" structures. To obtain the simulated data sets, random normal error was added to each of the 2n coordinates in the manner described by Young (1970). Specifically, an independent random normal deviate with variance σ_k^2 was added to each coordinate

$$x'_{ij} = x_{ij} + e_{ij},$$

where e_{ij} is $N(0,\sigma_k^2)$ and k is a subscript that indicates the error level. The error level introduced, σ_k, was specified as .0, .0225, .0625, .1225, .2225, .3225, and .5225. This method of adding error to the coordinates independently perturbs the distances.

In order to obtain the proximity matrix as an input matrix for the NMDS program, the interpoint distances measured according to the city-block metric were computed from the generated coordinates x'_{ij}; each coordinate having a different independent-random-error component. These error-perturbed distances were then submitted to the transformation

$$p_{ij} = 1.0/(d_{ij} + 1.0),$$

resulting in the set of simulated data p_{ij}. Three different random starts for HAP were employed for the simulated data for the ten-point structure and used to determine the affect of the different random starts on the resulting solutions of HAP. All of the k configurations generated by HAP as well as the random-starting configurations were input into the MINISSA (Roskam & Lingoes, 1970) multidimensional-scaling program.

The final solutions obtained from MINISSA using the k HAP (HAPIC), the random-starting (RIC), the Guttman-Lingoes (GLIC), and the Lingoes-Roskam (LRIC) initial configurations were compared to the "true" structure employing three measures: percent error (P.E.), the correlation between "true" and recovered distances - $r(d_t,d)$ (i. e., metric recovery), and stress. Since the purpose of NMDS is to find a set of numbers representing similarities between a set of point (i. e., the method attempts to provide a scaling solution with ratio properties), it is also of interest to index the degree of success in obtaining a ratio scaled solution by a measure of metric determinacy or metric recovery, the name give to the index of fit between the "true" and recovered configuration. Percent error is essentially a modification of Kruskal's stress and maybe used to compare two configurations (Kruskal, 1964; p.15):

$$P.E. = \sqrt{\frac{\sum_{i,j} (d_{ij} - a\hat{d}_{ij})^2}{\sum_{i,j} d_{ij}^2}} ,$$

where d_{ij} is the distance between objects in the "true" structure, $\hat{d}_{ij}$ is the distance between objects in the recovered configuration, and a is given by

$$a = \frac{\sum_{i,j} d_{ij}\hat{d}_{ij}}{\sum_{i,j} \hat{d}_{ij}^2} .$$

RESULTS

Observing the scaling results, presented in Table 1, for the ten-point configuration under error-free (σ_1), it appears, at least in terms of percent error and metric recovery, that the final configuration is very far from the "true" structure. This is somewhat surprising, since the literature suggests that, for the Euclidean metric, error-free data should scale close to the "true" structure (see Young, 1970; Spence, 1972). The results for the twenty-five-point configuration given in Table 2, except for the soluution obtained using a random start, show a close fit to the "true" structure for the error-free data (σ_1). In short, these preliminary observations on"perfect" data suggest caution in attributing assumed properties of NMDS in the Euclidean case to the city-block metric. Error-free data may scale well but "good" scaling can not be expected automatically and may depend heavily on the initial starts and the size and structure of the data set.

Using the data presented in Tables 1 and 2, it can be observed that within a particular error level, one of the HAP initial configurations leads to a final figure that has the lowest percent error. The percent-error values vary extensively within the two principal-component initial-configuration procedures, GLIC and LRIC. The random initial configuration generally has the higher percent error, which supports the conclusions given by Lingoes ans Roskam (1973; p. 69). As expected, an increase in the deterioration of fit of the final solution was observed as the amount of error increases for all of the procedures used to generate initial configuration.

Both percent error and the index of metric recovery confirm that the higher levels of error produce NMDS solutions that have bad fits with respect to the "true" structure, and also, that random initial configurations yield poor solutions compared to the results using rational initial starts, i. e., using GLIC, LRIC, and HAPIC. Comparisons of final solutions to the "true" structure

revealed that the final figures for the error-free data and small amounts of error (σ_1-σ_3) were generally good representations of the underlying structure in that the clusters of points were intact. Many of the final solutions for the ten-point configuration did not have the shape of the "true" structure; and in fact, only three of the HAP initial configurations resulted in the appropriate "kite" figure and these had the lowest percent-error values. None of the random or principal-component initial configurations resulted in the appropriate final figure. As larger amounts of error were added, fewer of the initial-configuration procedures resulted in final solutions with intact clusters. Clusters were either out of order or overlapped, thereby making substantive interpretation impossible or else ambiguous.

When the error-free simulated data for the twenty-five-point figure were scaled, however, all but one of the recovered configurations (the one resulting from a random initial start) corresponded to the original structure. The results correspond to the Young-Shepard observations in the Euclidean case, i. e., that smaller stress, smaller percent-error, and larger metric recovery values will be obtained for known configurations defined by a larger number of objects.

With small amount of error (σ_2), all but two of the final solutions had the same structure as the "true" configuration. The random initial configuration again resulted in a severe distortion, while the first HAP initial configuration resulted in a final solution that had the clusters intact, although not in the "correct" shape. The points within the clusters deviate more from their "true" positions as more error (σ_3, σ_4) is added, but only at the extreme levels (σ_6, σ_7) do the clusters become distorted and overlap. Unlike the results for the ten-point configuration, substantive interpretation is still possible with small and moderate amounts of error.

Relying solely on stress as a measure of fit would lead to misleading and erroneous results. It may be recalled that stress indicates how well the final solution fits the original input matrix of similarities and not, as is the case with the percent-error and metric-recovery indices, how well the final solution fits the "true" configuration. In fact, since stress values of .001 are not uncommon with high levels of error (see Tables 1 and 2, error levels σ_4 thru σ_6), Shepard's and Sherman's concern that stress is not a good measure of fit in the city-block metric is justified. For the high levels of error, the percent-error and metric-recovery indices of fit as well as an inspection of the final solutions show that these figures are not a good representation of the underlying structure

of the data.

The stress measure has one distinct advantage, however, since it can be used to determine how serious the problem of local minima is for the city-block metric. Local minima were identified by comparing the lowest stress value in an error level to all other stress values in the same error level. Lingoes and Roskam (1973) argue that a difference in stress as small as .01 could definitely be identified as a local minima, although a difference of this magnitude can be considered as "a minor local minima." Differences larger than .025 are considered "major local minimum." Table 4 contains the proportion of major and minor local minima for each error level for the ten- and twenty-five-point configurations. For the ten-point structure with no error, 95 percent of the final solutions are local minima, and only one error level has less than 80 percent local minima (σ_4). Similar results are apparent for the twenty-five-point structure as well. With no error, there are few local minima (12.5 percent), but as the amount of error increases, the number of local minima also increases. The large proportion of local minima suggests that the NMDS procedure is not able to obtain a solution that is an absolute minimum in the city-block metric. In summary, the problem of local minima seems to be inherent in the use of the city-block metric. When the results of the city-block-stress values are compared with the Euclidean-stress values (see Table 3) for the same ten-point configuration data, local-minima solutions are found only for random initial starts; none of the rational starting configurations produced a scaling solution that was a local minima.

In summary, the results of scaling in the city-block metric correspond to those results of the Euclidean metric in three ways: first, as the number of points increases, the degree of metric determinacy increases; second, as the error increases, the degree of metric determinacy decreases; and third, rational initial configurations yield "better fitting" final solutions than do randon initial configurations. The problem of local minima is generally more severe in the city-block metric, since the underlying structure is nonrecoverable from a city-block NMDS solution. Unfortunately, if the scaling is performed in the city-block metric, there is no way for the researcher to know if the scaling is a good and accurate representation of the underlying structure in the data. When scaling in the Euclidean metric, stress is usually a fairly good indicator of fit, but as the results discussed above show, stress is far from a perfect indicator of fit in the city-block context. Correlation between percent error and metric-recovery values is extremely high, a value of .98 (percent error with $1-r(d_t,d)$) was found for the Pearson product-moment correlation over all error levels and for both city-block configurations. Correlations between percent error, metric recovery and stress, however, were low; values of .55 (percent error and

stress), and .53 ($1-r(d_t,d)$ and stress) were obtained for the Pearson product-moment correlations over all error levels for the city-block configurations.

In order to insure that the rather unfortunate results obtained in the city-block metric would not be obtained for the more common Euclidean distance function, simulated data sets were first generated using a ten-point kite configuration under six levels of error and then under no error. These seven data sets were scaled by the MINISSA program in the Euclidean metric using initial configurations obtained by Guttman-Lingoes and Lingoes-Roskam schemes and by fifteen starting figures obtained from three random starts for the HAP program.

The performance of the various initial-configuration procedures in obtaining a final solution in the Euclidean metric is summarized in Table 3. With little or no error, the various indices indicate that the final figures are all close to the "true" structure in fit. The random procedure still does the worst compared to the rational initial configurations, but the difference does not appear to be of practical importance. Local minima solutions were found only when random-initial starts were employed (see Table 3), so local minima are not a serious problem in the Euclidean metric as they were in the city-block metric for this ten-point configuration. With no error, the final figures, for the most part, resembled a kite shape, and only small deviations of the points from their correct positions occurred. Although the deviations of the points became more pronounced for increasing levels of error ($\sigma_2 - \sigma_4$), the clusters remained intact. The larger amounts of error, however, produced clusters that were out of order, and for the largest amount of error (σ_7), the clusters overlapped. It may be recalled that this degeneration of the kite-structure occurred in the city-block metric with error-free data, since only three of the twenty final solutions had a "kite" shape.

In short, HAP appears competitive with the existing principal-component procedure and better than the random procedure. In all but one error level, a better final solution was obtained when HAP was used. Apparently, since HAP does not cause an increase in the number of local minima in the Euclidean metric, our previous finding that scaling in the city-block metric results in many local minima must be attributed directly to the metric used and not to the initial-configuration procedure.

One final comment about the Euclidean metric is in order. Young (1970) has argued that since stress is greater for a large rather than a small number of points, stress values should be high with large numbers of stimuli even though the reconstructed configuration could more accurately describe the structure underlying the data. For

our case in which the ten-point configuration was scaled in the Euclidean metric, the opposite result was found. In particular, while the indices of metric recovery suggested poor fit to the "true" figure, stress remained low indicating a good recovery of the data. Correlations between percent error, metric determinacy, and stress, computed over all levels of error, were all close to zero (i. e., .02 between percent error and stress and .06 between $1-r(d_t,d)$ and stress). Even though only one configuration was scaled in two dimensions, these results provide evidence that stress is not a good indicator of how well the underlying structure of the data has been recovered by the nonmetric-scaling procedure.

Since the city-block metric is not a stable metric in which to perform multidimensional scaling and since the problem of local minima is hidden, an experimenter does not know if a final scaling solution obtained in this metric represents the best obtainable. Further investigation into this problem in an applied situation was undertaken when actual data from experiments by Evans (1970) and by Lund (1970), which should have an underlying structure un the city-block metric, were scaled in the city-block and Euclidean metrics and were clustered using complete-link hierarchical clustering.

Evans (1970) used twenty-four complex hexagons in an experiment in which subjects were asked to free-sort thesestimuli into categories based on whatever classification scheme they thought appropriate. Because the hexagons can be considered analyzable stimuli, the summary proximity measures between the forms obtained from the sorts should have an underlying representation in terms of the city-block metric. Using a principal factors solution, Evans identified eight categories that could be used to structure the stimuli. In the reanalysis of the Evans data, complete-link hierarchical-clustering and multidimensional-scaling solutions were obtained, the latter using the HAP initial configuration in the city-block metric as well as the "canned" principal-component options in the published programs. The actual HAP configuration prior to scaling as well as the NMDS scaling solution with the HAP start and the clustering solution identified a three-group partition. On the other hand, the city-block solutions resulting from the principal-component initial configuration as well as the Euclidean NMDS solutions obtained by using both HAP and the principal-component initial configurations trapped a three-object group ({3,20,22}) away from the remaining objects in the partition containing it, as defined by the three-group partition determined by clustering. Typically, if the researcher uses a technique such as clustering for exploring the underlying structure

in a data set, and if the solution is consistent with the NMDS results, the researcher generally has increased confidence in the results of the scaling; particularly, when the nonrobust city-block metric is employed. In this example, however, even though the clustering results agree with the final solution resulting from a HAP start in the city-block metric and with the HAP start prior to the secondary scaling, the other procedures for generating initial configurations in the city-block and Euclidean metrics did not lead to such consistency. Thus a researcher is faced with a possible arbitrary choice as to which solution best describes the structure in the data.

The second applied example from Lund (1970) also deals with geometric stimuli, where again theory would suggest a city-block metric underlying the structure of the data. The nine Lund stimuli varied along two dimensions - three sets of three stimuli each had the same height and three sets of stimuli had the same roundness. The latter sets consisted of one member from each of the height categories. Using a similarity matrix obtained by the complete method of triads, Lund carried out a principal axis method and nonmetric scaling using Kruskal's M-D-SCAL program. The correspondence between the solutions was very close, with the principal axis solution indicating that two factors were sufficient toaccount for the data structure. In the NMDS scaling, the projections on the X axis ordered the stimuli according to the roundness dimension (1-4-7-2-8-5-3-6-9); the projections on the Y axis ordered the stimuli according to the height dimension (1-3-2-6-5-4-7-8-9). Using the same type of projection scheme, these orderings also were produced by the MINISSA program in the city-block metric employing GLIC, LRIC, and HAPIC. Thus for the Lund data set, scaling in the Euclidean or in the city-block metric with initial configurations generated by HAP or principal-component procedures yielded the same interpretation; in fact, these same results were found for a principal-axis metric procedure. Given such consistency, the researcher can be rather confident that the underlying structure in the data accounted for the clearness of the scalings and not the capriousness of the various techniques.

In summary, the data from these two experiments provide examples of what may occur when scaling in the city-block metric. Specifically, the underlying structure in the Lund data was revealed both by using metric and nonmetric procedures and by using either the city-block or the Euclidean distance function in the nonmetric procedure. On the other hand, the structure identified in the Evans data with the clustering strategy and also with the city-block final solution obtained by the MINISSA program using a HAP initial configuration, does not agree with the results

found in the Euclidean metric. Thus in this latter case, substantive interpretation may depend more on the technique used rather than the structure inherent in the actual data. What is needed is some taxonomy of situations in which the Euclidean scaling will accurately reveal the city-block structure whenever present. Until such a taxonomy is available, researchers should try a number of available procedures and hope that the interpretations that result are consistent. If the various techniques lead to convergent conclusions, as in the case of the Lund data, then the researcher may feel confident that indeed the underlying structure has been identified. Consequently, whenever convergent results are not procured, the researcher will have some arbitrariness in the choice of procedure and hopefully can chose on the basis of substantive considerations.

Table 1

Ten-Point Configuration (City-Block)

Percent Error

	σ_1=0	σ_2=.0225	σ_3=.0625	σ_4=.1225	σ_5=.2225	σ_6=.3225	σ_7=.5225
GLIC	.232	.186	.193	.204	.315	.336	.535
LRIC	.178	.171	.152	.189	.314	.334	.534
RIC	.191-.210	.216-.472	.312-.369	.302-.426	.30[illegible]-.360	.340-.514	.540-.579
HAP IC	.082-.276	.135-.200	.136-.245	.183-.250	.301-.417	.297-.586	.528-.575

$1-r(d,d_t)$

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.086	.086	.083	.284	.104	.320	.881
LRIC	.055	.059	.059	.281	.111	.314	.902
RIC	.064-.085	.102-.788	.265-.408	.249-.369	.270-.481	.324-.859	.861-.875
HAP IC	.015-.217	.065-.082	.048-.094	.230-.461	.096-.151	.230-.999	.842-.922

Stress

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.119	.129	.092	.107	.029	.001	.092
LRIC	.085	.109	.024	.001	.029	.013	.057
RIC	.094-.113	.133-.307	.191-.255	.246-.284	.085-.196	.001-.261	.117-.347
HAP IC	.057-.213	.065-.129	.062-.125	.001-.146	.034-.218	.061-.305	.067-.178

Table 2

Twenty-Five-Point Configuration (City Block)

Percent Error

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.014	.032	.170	.172	.302	.302	.358
LRIC	.013	.032	.076	.205	.321	.302	.353
RIC	.483	.283	.076	.444	.319	.519	.483
HAP IC	.012-.103	.032-.195	.076-.169	.171-.261	.295-.316	.342-.422	.355-.358

$1-r(d,d_t)$

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.001	.003	.078	.088	.255	.271	.349
LRIC	.001	.003	.017	.119	.292	.272	.350
RIC	.728	.220	.017	.591	.275	.817	.674
HAP IC	.001	.003-.105	.017-.077	.087-.236	.246-.275	.339-.489	.350-.354

Stress

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.0008	.0008	.1392	.0007	.0248	.0007	.0131
LRIC	.0007	.0007	.0008	.1041	.1124	.0008	.0700
RIC	.3725	.2614	.0010	.3404	.1196	.3518	.3367
HAP IC	.0008	.003-.261	.001-.142	.001-.138	.001-.117	.179-.301	.001-.070

Table 3

Ten-Point Configuration (Euclidean)

Percent Error

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.039	.048	.093	.155	.251	.303	.429
LRIC	.041	.048	.094	.167	.254	.313	.431
RIC	.034-.099	.045-.089	.087-.114	.161-.185	.247-.259	.299-.358	.422-.433
HAP IC	.019-.037	.039-.053	.080-.114	.151-.181	.245-.255	.292-.358	.422-.433

$1-r(d,d_t)$

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.004	.005	.021	.057	.147	.216	.490
LRIC	.004	.006	.021	.067	.153	.233	.495
RIC	.001-.020	.004-.018	.018-.027	.063-.077	.141-.160	.189-.217	.476-.502
HAP IC	.001-.003	.004-.007	.015-.028	.054-.074	.141-.149	.188-.217	.474-.502

Stress

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
GLIC	.0008	.0006	.0007	.0007	.0006	.0007	.0007
LRIC	.0008	.0006	.0007	.0005	.0006	.0008	.0007
RIC	.0007-.0465	.0006-.051	.0007-.063	.0005-.001	.0006-.0008	.0007-.0009	.0007
HAP IC	.0007-.0008	.0006-.0007	.0006-.0007	.0006-.0008	.0006-.0007	.0006-.0009	.0006-.0007

Table 4

Proportion of Minor and Major Local Minima in the City-Block Metric

Ten-Point Configuration

Error-level	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
Minor Local Minima	5.0*	0.0	0.0	0.0	5.0	17.6	0.0
Major Local Minima	90.0	95.0	90.0	65.0	85.0	66.7	95.0

Twenty-Five Point Configuration

Error-level	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
Minor Local Minima	0.0*	0.0	0.0	0.0	50.0	0.0	0.0
Major Local Minima	12.5	50.0	37.5	37.5	37.5	75.0	50.0

Proportion of Minor and Major Local Minima in the Euclidean Metric

Ten-Point Configuration

Error-level	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
Minor Local Minima	0.0*	0.0	0.0	0.0	0.0	0.0	0.0
Major Local Minima	5.0	5.0	5.0	0.0	0.0	0.0	0.0

* indicates proportion of local minima

REFERENCES

Arnold, J. B. A multidimensional scaling study of semantic distance. Journal of Experimental Psychology Monograph, 1971, 90, 349-372.

Attneave, F. Dimensions of similarity. American Journal of Psychology, 1950, 63, 516-556.

Behrman, B. W., and Brown, D. R. Multidimensional scaling of form: a psychophysical analysis. Perception and Psychophysics, 1969, 4, 19-28.

Brown, D. R., and Andrews, M. H. Visual for discrimination: Multidimensional analyses. Perception and Psychophysics, 1968, 3, 401-406.

Brown, D. R., and Brumaghim, S. H. Perceptual equivalence, pattern perception, and multidimensional methods. Perception and Psychophysics, 1969, 4, 253-256.

Evans, G. T. The analysis of categorizing behavior. Psychometrika, 1970, 35, 367-392.

Handel, S., and Imai, S. The free classification of analyzable and unanayzable stimuli. Perceptionand Psychophysics, 1972, 12, 108-116.

Hyman, R., and Well, A. Judgments of similarity and spatial models. Perception and Psychophysics, 1967, 2, 233-248.

Hyman, R., and Well, A. Perceptual separability and spatial models. Perception and Psychophysics, 1968, 3, 161-165.

Kruskal, J. B. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. Psychometrika, 1964, 29, 1-27.

Landahl, H. D. Neural mechanisms for the concepts of difference and similarity. Bulletin of Mathematical Biophysics, 1945, 7, 83-88.

Lingoes, J. C., and Roskam, E. E. A mathematical and empirical analysis of two multidimensional scaling algorithms. Psychometrika Monograph Supplement, 1973, 38.

Lund, T. Multidimensional scaling of goemetrical figures. Scandinavian Journal of Psychology, 1970, 11, 246-254.

Rapoport, A., and Fillenbaum, S. An experimental study of semantic structures. In A. K. Romney, R. N. Shepard, and S. B. Nerlove (Ed.), Multidimensional scaling: Theory and applications in the behavioral sciences, v. II/ Applications. New York: Seminar Press, 1972, pp. 96-133.

Roskam, E. E., and Lingoes, J. C. MINISSA-I: A Fortran IV (G) program for the smallest space analysis for square symmetric matrices. Behavioral Sciences, 1970, 15, 204-205.

Shepard, R. N. Metric structures in ordinal data. Journal of Mathematical Psychology, 1966, 3, 287-315.

Shepard, R. N. Representation of structure in similarity data: Problems and prospects. Psychometrika, 1974, 39, 373-421.